Quantum Computing-based Airline Crew Rostering Optimization

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1.
Executive Summary

Across the globe, the airline industry deploys a suite of optimization tools for efficient, speedy and cost-effective flight scheduling, fleet assignment, aircraft maintenance and crew scheduling. With the increased availability of quantum computing-based optimization solutions, these high-impact tools can be upgraded to deliver superior performance in both run-time and solution quality for large-scale problems.

This paper explores how quantum optimization-driven crew roster designs for airline operations can deliver an equitable distribution of work and run-time reduction at scale when compared to open-source classical optimization routines.

2.
Background

IATA (International Air Travel Association) and ICAO (International Civil Aviation Organization) are leading groups in the aviation and airline industry, which have assigned airline codes to more than 5,000 local, regional and international airlines. Prior to the COVID-19 crisis, 1,126 commercial airlines operated more than 16 million flights, carrying 4.5 billion customers annually [1]. American Airlines, a major US airline, operates approximately 5900 flights per day (translating to ~70000 flights per month) with a ~900-strong fleet size [2].

Running airline operations smoothly and cost effectively on such a massive scale requires intricate planning and scheduling. Airline companies frequently make use of optimization tools to achieve this end in the areas of flight scheduling, fleet assignment, aircraft maintenance and crew scheduling.

Specifically, airline crew scheduling is a computationally intensive problem with large datasets and numerous complex constraints. It is typically solved sequentially as a two-part problem - crew/flight pairing, followed by crew rostering.

- **Crew/flight pairing:** This takes the list of flight legs for a defined period, and sequences them together, each of which is referred to as a ‘crew/flight pair’.
- **Crew rostering:** Crew rostering optimization uses flight pairings generated by flight pairing optimization, along with crew details such as base location and crew count and assigns a pre-defined number of crew members to each pairing to create a ‘crew roster’.

In this paper, we will address the crew rostering problem with a focus on creating satisfaction for aviation authorities, labor unions and airline operations and profitability, rather than on optimal cost allocation. This is achieved by the efficient allocation of flights to crew members, leading to equitable distribution of work, which has a positive direct impact on employee satisfaction and airline operational performance, and indirectly on retention rates, costs and brand value.

The paper focuses on designing crew rosters using quantum annealer-based optimization routines. The complexity of crew rostering generally increases with an increase in crew size and the number of flights. In comparison to current classical optimization solutions, quantum computing-based optimization algorithms allow a more effective search of the solution space to generate better results in a shorter time when run at scale for planning and scheduling.

3.
Business Drivers and Challenges

Some of the major complexities in planning optimal crew rosters include:

- Increasing complexity of crew rostering with an increasing number of flights and subsequently increasing number of flight pairing and crew size
- Constraints of federal authorities that bind airlines, such as maximum and minimum flying hours, rest period between assigned flight pairings, minimum number of assigned crew members for each flight pairing and cost of operations among others
- Real-time planning, a necessary component to handle unforeseen situations, such as flight delays, unavailability of crew members and rescheduling of crew member duties to minimally impact subsequent schedules of other crew members

Crew rostering is a planning problem, and calls for the design of an optimal schedule for each crew member for a pre-defined airline operational period. Key constraints which must be taken into consideration while formulating the problem include:

- Binding the minimum and maximum flying hours for crew members by a lower and upper limit
- Assigning each flight pair to a specified number of crew members
- Assigning duties to each crew member
- Providing pre-defined rest time in the duty schedule of crew members between consecutively assigned flights
- Ensuring that crew members are not assigned more than one flight pairing at the same time

4.
Problem Formulation

In formulating the required objectives, crew rostering takes into account flight schedules, number of crew members and bases, activity rosters and constraints. We describe our approach to problem formulation as follows:

Decision variable

Decision variables are binary variables that indicate the assignment of every crew at a base to every pairing starting from that base. For example, the decision variable X_{cp} represents that the c^{th} crew is assigned to the p^{th} pairing. Given n crew members at a base and k pairings, there will subsequently be $(n*k)$ binary decision variables.

Constraints

- Base of pairing and crew should be the same, as this would reduce the traveling cost of a crew member from their base to the pairing base, leading to a cost-effective solution. A matrix B_{cp} is formulated in which each element is 1 or 0. If the crew base and pairing base are the same, then it would be 1, otherwise 0.
- Total flying hours should be within the prescribed limit defined by Federal Aviation Regulations (FAR). This does not include rest time between the flight legs.

$$\sum_{p=1}^{k} B_{cp} \times f_p \times X_{cp} \geq f_{min} \; \forall c = 1,2,\dots,n$$

$$\sum_{p}^{k} B_{cp} \times f_p \times X_{cp} \leq f_{max} \; \forall c = 1,2,\dots,n$$

X_{cp}: Binary variable, equal to 1, if p^{th} pairing is assigned to c^{th} crew member, otherwise 0

B_{cp}: Binary variable, equal to 1, if crew base and pairing base is same, otherwise 0

f_p: Flying time of pairing p

f_{min}: Minimum flying hours for a crew member

f_{max}: Maximum flying hours for a crew member

n: Total number of crew members

k: Total number of pairings

- Overall flying hours consist of flying hours with rest time. It should be within the prescribed limit.

$$\sum_{p=1}^{k} B_{cp} \times O_p \times X_{cp} \leq O_{max} \quad \forall c = 1,2,\dots,n$$

$$\sum_{p=1}^{k} B_{cp} \times O_p \times X_{cp} \geq O_{min} \quad \forall c = 1,2,\dots,n$$

O_p: Overall flying hour of p^{th} pairing

O_{max}: Maximum allowable overall flying hours for a crew member

O_{min}: Minimum allowable overall flying hours for a crew member

- Inter-pairing time, which is the time between two consecutive duties of a crew member. This should be more than a user-defined minimum time.

$$B_{cp} \times X_{cp} + B_{cp+1} \times X_{cp+1} \leq 1 \qquad d_{p+1} - a_p < t_{min}$$

$d_{(p+1)}$: Departure time of $(p+1)^{th}$ pairing

a_p: Arrival time of p^{th} pairing

t_{min}: Minimum inter-pairing time

- Each pairing should be assigned a minimum number of crew members. This constraint ensures that all pairings have crew members.

$$\sum_{c=1}^{n} B_{cp} \times X_{cp} = C_{min} \quad \forall p = 1,\ 2,\ 3, \ldots, k$$

C_{min}: Minimum number of required crew

Objective

To minimize total cost incurred on the airline due to allocation of different pairings.

$$Min\left(\sum_{c=1}^{n}\sum_{p=1}^{k} B_{cp} \times X_{cp} \times r_c\right)$$

r_c: Cost of c^{th} crew member

5.
Solution Methodology

Our current work explores the application of quantum annealer-based optimization for crew rostering along with a comparison to an open-source python-based classical optimizer.

Solving any optimization problem requires two steps - one, formulating the problem, and two, obtaining the optimal solution to the formulation. The first step constitutes understanding the problem and formulating it in mathematical terms. This mathematical formulation can be done in several ways, such as Linear Programming (LP), Mixed Integer Linear Programming (MILP), non-linear and quadratic. Based on the convenience of formulating the problem and the availability of algorithms, techniques and tools to solve the model, a method of formulation is chosen. Once the problem is formulated, the second step in the optimization problem is obtaining the solution with the most optimal cost. As the problem size increases, the problem cannot be solved analytically, and brute force methods theoretically could take exponentially longer. Hence, numerical approaches are utilized to provide approximate solutions to large optimization problems.

Classical optimization-based approach

We have formulated the problem as a binary linear programming one. Classical LP solvers have been used for decades to solve optimization problems for industry and academia. For crew rostering, the open-source python library ‘Pyomo’ has been utilized to model the optimization problem. Pyomo can be used to define abstract problems, create concrete problem instances and solve these instances with standard classical optimization routines. It provides a capability that is commonly associated with algebraic modeling languages like AMPL and GAMS. Pyomo leverages the capabilities of the Coopr software, which integrates Python packages for defining optimizers, modeling optimization applications and managing computational experiments. COIN-OR Branch and Cut (CBC) solver has been used to solve the problem. It is an open-source mixed-integer linear programming solver.

Quantum annealer-based approach

Quantum annealers are essentially ISING machines that help solve combinatorial optimization problems. Solving optimization problems with quantum annealers requires encoding industry and academia problems to energy minimization problems. Quantum annealers employ energy encoding to map problems to hardware and follow a nature-inspired quantum optimization paradigm. It allows the system to evolve through time while maintaining control over the pace of evolution, and when given enough time, a system will achieve its lowest energy point.

While classical algorithms, such as simulated annealing, also employ a similar phenomenon, quantum annealers can deliver significant performance and quality improvement over classical algorithms using quantum mechanical phenomena such as quantum tunneling.

In the context of crew rostering, the complexity of problems increases with an increase in the number of flights, crew members and flight bases - which, in turn, increases the number of flight pairings. Current classical optimization systems are developed over decades and provide good approximations for medium-sized problems. However, the run-time increases substantially with an increase in problem size and a decrease in result quality. Quantum solvers such as hybrid classical-quantum solvers on quantum annealers and quantum-inspired classical optimization algorithms (QIO) can improve solution quality while reducing the run-time for certain types of energy landscapes representing a particular class of problems.

D-Wave's quantum annealers currently support optimization models in the form of Constrained Quadratic Models (CQM) or Binary Quadratic Models (BQM) to define objectives and constraints. BQMs are further transformed into Quadratic Unconstrained Binary Optimization (QUBO) or equivalent ISING formulation in ferromagnetism.

To use quantum annealers, first, the optimization problem must be converted to CQMs or QUBOs. CQM, as the name suggests, are constrained models with binary or integer decision variables. QUBOs have only binary decision variables, and the constraints need to be converted into an unconstrained problem using the penalty method. In this method, constraints are added to the objective function with a penalty. If a solution fails to satisfy a constraint, then the corresponding penalty will be added to the total cost.

Dimod has been utilized for CQM formulation, while Qubovert has been utilized to formulate the QUBO model. D-Wave's Leap Hybrid Solvers, which are used to solve the problem, implement state-of-the-art classical algorithms together with intelligent allocation of the quantum computer to parts of the problem where it benefits most.

6.

Results & Discussion

Data description

Table 1 Dataset Description

Attributes		Base 1	Base 2
Dataset	Number of Pairings (n)	22	126
	Number of Crew Members (k)	10	30
	Problem Size (n*k)	220	3780
Constraints	Maximum Flying Hours	100	
	Maximum Overall Hours	300	
	Number of Crew Members/Pairing	2	
	Minimum Inter-pairing Time (hours)	18	

Results for base 1

Table 2 KPIs for Base 1

Attributes		Classical	CQM
Flying Hours	Minimum	53.6	58.93
	Maximum	95.05	80.43
	Mean	69.96	69.96
	Standard Deviation	14.32	7.77
Overall Hours	Minimum	159.38	180.28
	Maximum	292.25	272.55
	Mean	216.52	216.52
	Standard Deviation	50.37	32.16
Number of Pairings	Minimum	4	3
	Maximum	6	5
Number of Flights	Minimum	28	28
	Maximum	48	41
Average Inter-pairing Time (hrs)	Minimum	66.5	35.32
	Maximum	138.5	269.28
Total Run-time (seconds)		5.121	5.752

Results for base 2

Table 3 KPIs for Base 2

Attributes		Classical	CQM
Flying Hours	Minimum	57.76	47.5
	Maximum	93.2	97.33
	Mean	78.92	82.31
	Standard Deviation	10.25	10.60971
Overall Hours	Minimum	148.01	114.67
	Maximum	273.08	279.05
	Mean	206.17	215.89
	Standard Deviation	37.94	35.062
Number of Pairings	Minimum	6	5
	Maximum	12	12
Number of Flights	Minimum	32	31
	Maximum	59	55
Average Inter-pairing Time (hrs)	Minimum	50.82	28.60
	Maximum	68.07	104.38
Total Run-time (seconds)		77.235	55.037

Visualizations

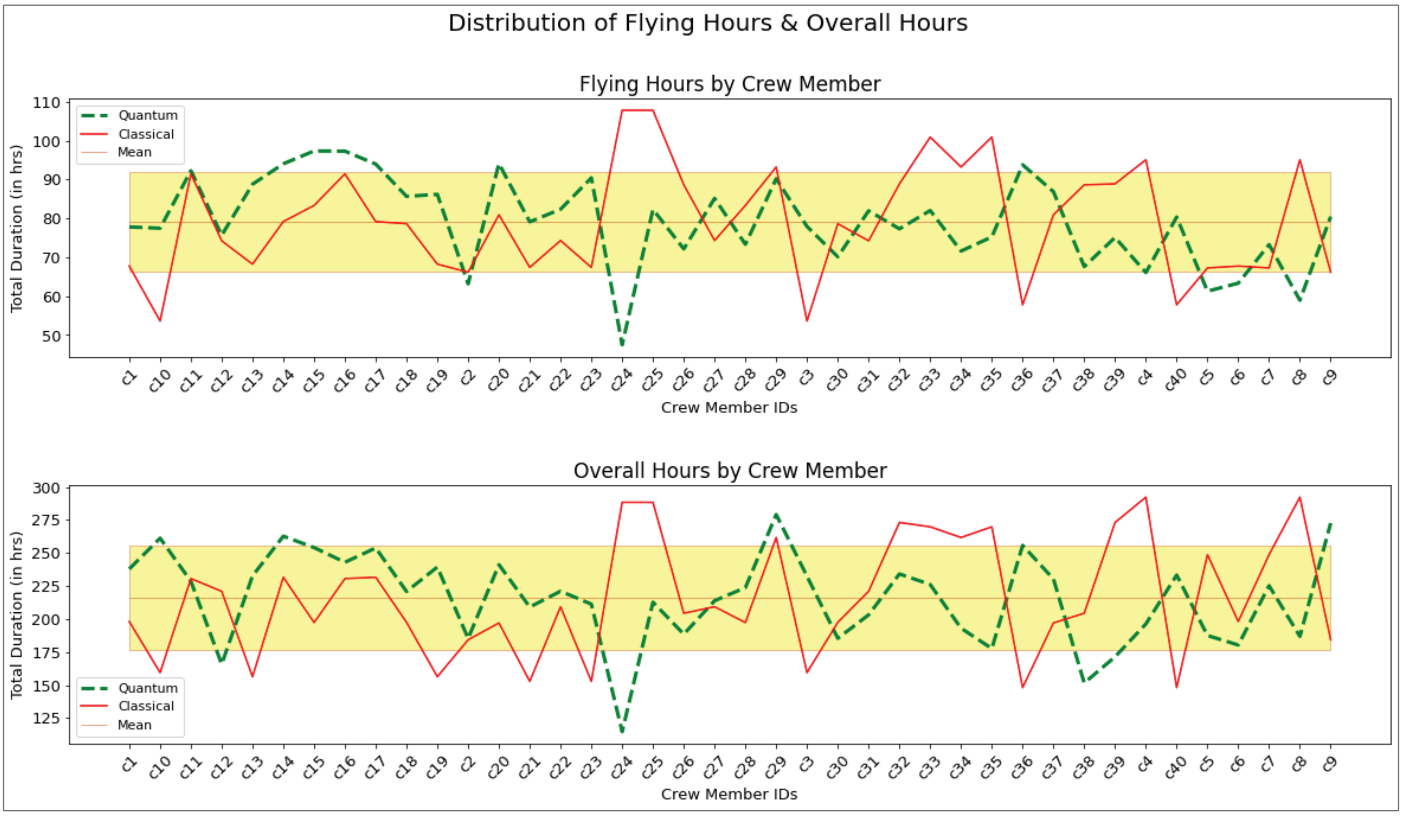

Figure 1 Distribution of Flying and Overall Hours Over Entire Crew Roster

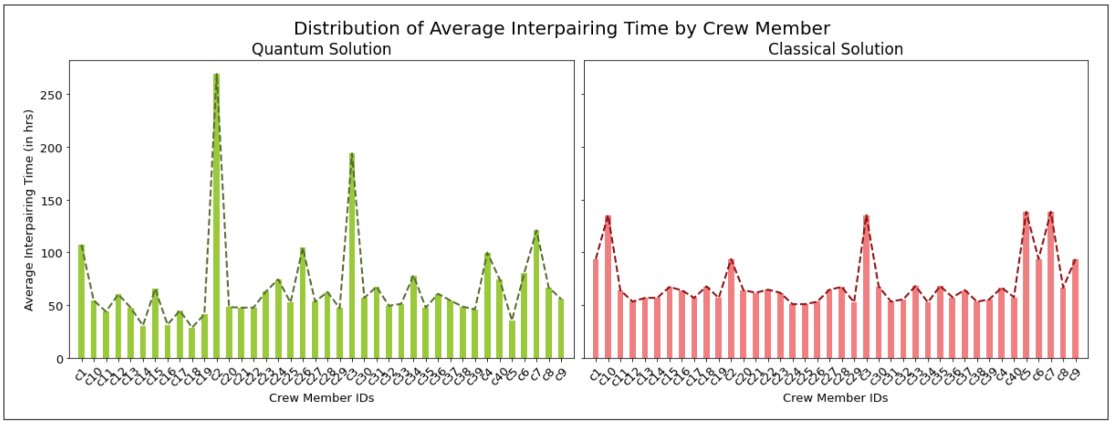

Figure 2 Distribution of Average Inter-pairing Time (in hours) Over Entire Crew Roster

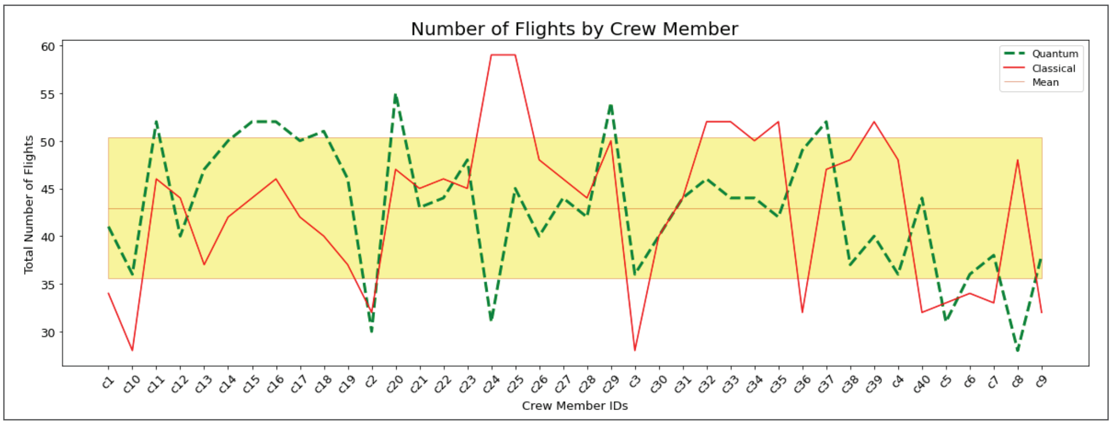

Figure 3 Total Number of Flights Over Entire Crew Roster

Analysis of results

Quality of Solution

- As shown by Figure 1, the quantum solution provides a relatively even distribution of flying hours and overall hours over the entire roster
- Tables 2 & 3 further support the above claim, as the standard deviation of flying hours and overall hours for the quantum solution is less than or equal to the classical solution
- The quantum solution also provides a more even distribution of the number of flights, as compared to the classical solution, as shown in Figure 3
- In terms of average inter-pairing time, as shown in Figure 2, the classical solution provides a relatively even distribution

Scalability of Solution

The CQM quantum annealer provides a scalable solution as proven by the results on different bases. We have observed that the solver time does not increase significantly with problem size while giving an acceptable quality of a valid solution. There is a slight increase of 0.7% in the total time as the problem size increased 17 times.

7.

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Siddharth Shankar is currently working as a Lead Data Scientist at Mphasis NEXT Labs. The role involves research into various topics such as Model Optimization, Agent-based Simulation and Quantum Machine Learning to develop innovative solutions with a clear business objective while leveraging Cloud services and the principles of MLOPs.

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Rohit has over 13 years of experience including more than 7 years in Data Science. He is leading the Quantum Computing initiative at Mphasis NEXT Labs for 3+ years. He holds a Bachelor's degree in Electronics & Communication Engineering and Post Graduate Diploma in Financial Management. He has ideated, designed, developed and managed AI & Quantum computing based solutions and client projects in areas of BFSI, Logistics, Life Sciences and Process Optimization, among others.

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