Quantum Annealer-based Portfolio Optimization

Whitepaper by Rohit Patel, AVP – Data Science and Quantum Computing |
Ashutosh Vyas, Senior Data Science Manager – Mphasis NEXT Labs | Narayan Mishra, Assistant Manager – Data Science

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1.
Introduction

Financial analysis is a critical tool to optimize investments in securities. It directly affects the quality of investment decisions made by an entity. One such area of financial analysis is portfolio optimization, where the investors for a given budget, are suggested a subset of investment options to maximize return while minimizing risk. Different investors have different risk appetites, which require personalization of the recommended portfolios for a particular type of investor. For example, generally, investors in a lower age bracket tend to be less risk-averse than investors in a higher age bracket.

The concept of portfolio optimization essentially involves return maximization while ensuring minimization of risk with the given set of securities across or within different asset classes. Modern portfolio theory, pioneered by American economist Harry Markowitz, provides a method to realize the goal of portfolio optimization mentioned above. It builds on the idea of diversification, where securities should be viewed from the point of interdependence of overall risk and return rather than looking at each of them in isolation. It takes the weighted sum of average returns of individual assets to calculate portfolio return and derives portfolio risk as a function of variance in return of each asset and correlations between a pair of assets.

Using the methodology presented by modern portfolio theory, one can generate multiple risk-return scenarios by varying the weighted allocation of budget across a subset of investment options from a given set. Plotting these different combinations on a 2-dimensional graph with the portfolio's risk on X-axis and the portfolio's return on Y-axis, one can visualize the best risk-return combinations for investors of different risk appetites. By identifying and joining the points representing the most efficient portfolios from the above combinations, an upward sloping curve called 'efficient frontier' can be drawn. Any investment choice that falls underneath the curve is less optimal for a given portfolio risk.

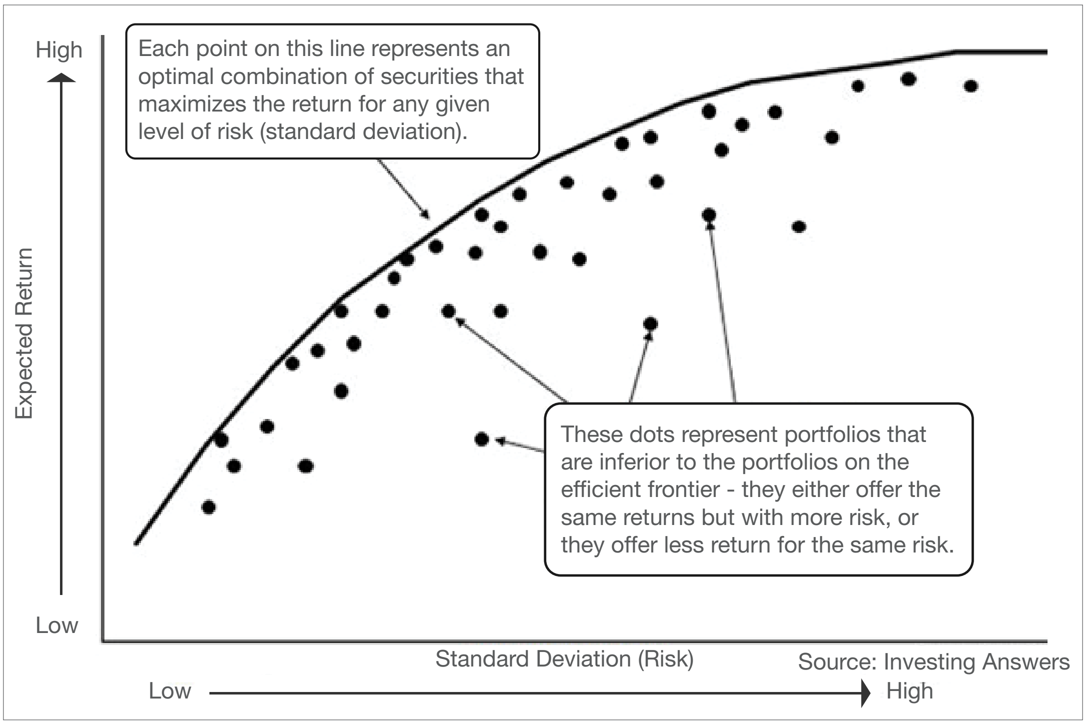


Figure 1: Efficient Frontier [3]

Based on the idea above, for a given risk level, optimal and suboptimal portfolios can be built based on the proportion of budget allocated for each of the securities under consideration. Optimal portfolio selection hence becomes an optimization problem where the aim is to identify the subset of securities and the proportion of budgets allocated to them to maximize the return for a given risk level. Portfolio optimization is a major research area where different heuristic and non-heuristic methods are formulated and solve the problem.

We here introduce the use of quantum computing, specifically quantum optimization, based approach to solve the portfolio optimization problem. Our approach makes use of quantum annealers and quantum-inspired optimization approaches to identify the right securities and distribution of budgets to build an optimal portfolio. Based on our experiments on stock data from DOW30, S&P500 and other indices, we have observed an improvement in performance with respect to execution time and portfolio quality.

2.
Business Objectives

The core objectives of portfolio optimization are as follows:

- **Maximizing return on investment:** Investors would always expect to get as much profit from an investment as possible given the risk appetite. Thus, the portfolio developer must keep this as an essential business objective.
- **Managing investor risk appetite:** Different investors have different needs and appetites for return to risk tolerance factor. The goal of portfolio selection hence is to identify the most optimal portfolio as per investor risk appetite.

3.
Key Factors to Build an Optimal Portfolio

The key factors that help in the development of a good investment portfolio are as follows:

- **Investment diversification:** Management of unsystematic risk or non-priced risk, investment portfolios should be extensively diversified. A diversified portfolio protects the investors from the risk of losing money if a single asset underperforms.
- **Identify the right investment opportunities:** New investment opportunities across asset classes should always be tracked and monitored to keep up with the changing market behavior. This helps in better diversification of the resources and provides high-profit opportunities.
- **Adapting to changing investment climate:** Portfolio optimization is driven by analysis of the historical performance of securities under consideration. Careful selection of historical data is important to understand the parameters of the securities market.

4.

Assumptions in Building Portfolios Using MPT

To hold true, Modern Portfolio Theory, on which the notion of portfolio optimization is founded, makes the following assumptions:

- **Frictionless markets:** One of the assumptions is that markets are frictionless, meaning that there are no transaction costs, limitations or other market restraints. In practice, this is frequently discovered to be untrue. Because there are frictions in the market, applying the current portfolio theory is more difficult.
- **Normal distribution:** The normal distribution of returns is another assumption in the current portfolio theory. When utilizing the return data as inputs, it ignores notions like skewness, kurtosis and so forth. The returns are frequently found to be non-normally distributed. The modern portfolio theory's violation of assumption makes it difficult to use once more.
- **Dynamic coefficients:** The data used in portfolio optimization contains few coefficients, such as the correlation coefficient. As market conditions change, the correlation coefficient may fluctuate. In some circumstances, the assumption that these coefficients remain constant may not be correct.
- **Volatile market**: Asset prices keep changing and this greatly impacts the assessment of someone investing in one specific asset for a long duration.

5.

Modeling Portfolio Optimization Problem

Portfolio optimization and asset allocation problem for a given set of investment opportunities with different risk-return characteristics have many possible candidate solutions. Identifying the optimum portfolio which has a perfect trade-off between higher portfolio returns and lower portfolio risk is a non-trivial task. We here propose a quantum annealer-based optimization approach to build optimized investment portfolios for investors with different risk appetites. The approach to solving this problem is mentioned as follows:

Calculate the annualized daily rate of return for each asset using the acquired daily price data of assets

↓

Calculate the annualized covariances between the daily rate of return values of all assets

*The variance of the portfolio is referred to as portfolio risk and the standard deviation of the portfolio is referred to as portfolio volatility

↓

Formulate optimization objective and constraints as QUBO/ISING models

↓

Run optimization solvers

In our experiments, we have considered 5 years of historical price data for stocks from DOW30, S&P500 and other market indices. Problem Formulation: Quantum annealer-based solvers require problems to be formulated as Binary Quadratic Models (BQM), specifically, ISING models or Quadratic Unconstrained Binary Optimization (QUBO) models.

In our portfolio optimization problem, the objective function has two parts:

- Maximize portfolio return
- Minimize portfolio risk

We can convert our multi-objective risk-return optimization problem, which is harder to formulate and solve, to a single objective optimization problem by considering financial ratios such as the Sharpe ratio or its derivatives. Sharpe ratio provides a measure of risk-adjusted return. Invented by Nobel prize winner William F. Sharpe, it is used to help investors comprehend the return on investment in relation to its risk.

In article [2], the Sharpe ratio is defined in simple terms as the ratio which adjusts performance to account for an investor's excessive risk. It describes how much excess return one can realize for the volatility of holding a riskier asset. An investor can use the Sharpe ratio to see if the investment meets his needs by comparing two different portfolios with the same risk or returns. It is defined as follows:

$$S_a = \frac{E[R_a - R_b]}{\sigma_a} \quad (1)$$

Where, S_a = Sharpe ratio

E = expected value

R_a = asset return

R_b = risk-free return

σ_a = standard deviation of the asset excess return

Though the Sharpe ratio combines risk-return in a single formula, it does not lend itself to a quadratic objective formulation which is a prerequisite for quantum annealers-based solvers such as D-wave Leap Hybrid Solvers. A derivative of the Sharpe ratio, though, **Chicago Quantum Net Score (CQNS)** can solve this problem and convert our objective to a quadratic form.

In arXiv:2007.01430, the authors reformulated and transformed the Sharpe ratio formula into a linear quadratic form as equation (2):

$$\ln(S_a) = \ln(E[R_a - R_b]) - \ln(\sigma_a) \quad (2)$$

However, because equation (2) is not a consistent quadratic form, the authors used the Chicago Quantum Net Scores (CQNS), which is provided by the equation:

$$CQNS(w;\alpha) = Var(R_w) - E[R_w]^{2+\alpha} \quad (3)$$

Where R_w is a weighted portfolio and $\alpha \epsilon R$.

The below section describes the mathematical formulation of the portfolio optimization problem.

Decision variables: The decision variables are $q_{i,k}$ which represents the k^{th} allocation in i^{th} asset. There are two types of constraints: budget constraints and asset allocation constraints. The budget constraints signify that the total investment in assets should not exceed the available budget. The allocation constraints are required to be satisfied for each asset. If q_{i_1,k_1} is 1, then the $q_{i,k}$ for all values of i and k should be 0 except for k_1.

$q_{i,k} \in \{0,1\}$ – Binary decision variable $q_{i,k}$ refers to the k^{th} part of the capital invested in stock *i*.

QUBO formulation: For QUBO formulation, the constraints optimization problems are converted into unconstraint optimization problems using the penalty method. In the penalty method, the constraints are added to the objective function by multiplying it with a penalty. The idea behind it is if the solution fails to satisfy the constraints, then the penalty will be added to the total cost.

Objective function: Minimize portfolio risk and maximize portfolio return (minimize CQNS)

$$Min \quad z = \sum_i \sum_j \left(\sum_k k \times q_{i,k}\right) \times \left(\sum_k k \times q_{j,k}\right) \times Cov(i,j) - \sum_i (r_i)^{(2+\alpha)} \left(\sum_k k \times q_{i,k}\right) \quad (4)$$

$$Min\ z = \sum_i \sum_j \left(\sum_k k \times q_{i,k}\right) \times \left(\sum_k k \times q_{j,k}\right) \times Cov(i,j) - \sum_i (r_i)^{(2+\alpha)} \left(\sum_k k \times q_{i,k}\right) \quad (5)$$

Constraints:

$\sum_i \sum_k k \text{ X } q_{i,k} \leq 1$, total capital invested in all assets individually should not exceed 100% of the capital available

$\sum_i q_{i,k} = 1\ \forall\ i$, for each single asset, only one capital option should be 1

Where,

- *i,j* : index to represent financial assets (e.g., stocks, etc.)
- Max portion of total capital that can be invested in single asset is 0.1 or 10%
  - $k \in \{0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1\}$
  - *k* – capital invested in a single asset
- $q_{i,k}$: Binary decision variable refers the k^{th} part of the budget invested in the i^{th} stock, $\in \{0,1\}$
- r_i – annualized daily rate of return of single asset *i*
- ***Cov (i,j)*** **–** Covariance of annualized daily returns among assets *i* and *j*

α – parameter to leverage the effect of portfolio return in the objective function

Quadratic Unconstrained Binary Optimization (QUBO)

$$Min\ z = M \times \Bigg(\sum_i \sum_j \left(\sum_k k \times q_{i,k} \right. \times \left(\sum_k k \times q_{j,k}\right) \times Cov(i,j)$$
$$- \sum_i (r_i)^{(2+\alpha)} \left(\sum_k k \times q_{i,k}\right)\Bigg)$$
$$+ A \times \left(\sum_i \sum_k k \times q_{i,k} - 1\right)^2 + B \times \sum_i \left(\sum_k q_{i,k} - 1\right)^2 \quad (6)$$

Where,

- A, B are large penalties for the constraints
- *M* - parameter to leverage the effect of the objective function in QUBO and to compensate for the change in values of the rate of return of single assets
- *α* - parameter to leverage the effect of portfolio return in the objective function

Why Quantum for Portfolio Optimization

Any optimization problem is solved in two steps. The first step is to comprehend the problem and formulate a mathematical solution. Linear Programming (LP) formulation, Mixed Integer Linear Programming (MILP) formulation, non-linear formulation and quadratic formulation are all examples of problem formulation. The convenience of defining the problem, as well as the availability of algorithms, methodologies and instruments to solve the formulation, are used to choose a formulation approach. The formulation's second stage is to obtain the solution at the lowest possible cost. Numerical approaches are employed to provide approximate answers since large optimization problems cannot be solved analytically, and brute force methods might theoretically take exponentially longer as the input size grows.

Optimization problems can be transformed into energy minimization problems, which quantum annealers can solve. Quantum annealers employ energy encoding to map problems to hardware and follow a nature-inspired quantum optimization paradigm. It allows the system to evolve through time while maintaining control over the pace of evolution, and when given enough time, a system will achieve its lowest energy point. Such phenomenon has been captured as part of classical algorithms such as Simulated Annealing. Quantum annealers deliver a performance and quality improvement of such classical algorithms by utilizing quantum mechanical phenomena like quantum tunneling for large optimization problems. Quantum annealers define issue objectives and constraints using Binary Quadratic Models (BQM), which are then transformed into Quadratic Unconstrained Binary Optimization (QUBO) or analogous ISING formulations in ferromagnetism.

In real-world market circumstances, the pool of financial assets contains a wide range of assets from which we must select assets for our portfolio. As can be seen, many funds contain more than 50 assets and allowing additional flexibility in portfolio creation will result in a portfolio with a large number of decision variables to be optimized. For example, S&P500 index has 500 assets and if we have buckets of 1% for each asset allocation, we have 101 buckets for each asset and in total, we will have 50500 (= 500 x 101) decision variables to be optimized. For a high number of variables, standard portfolio optimization solutions such as Monte Carlo simulation may deliver lower performance.

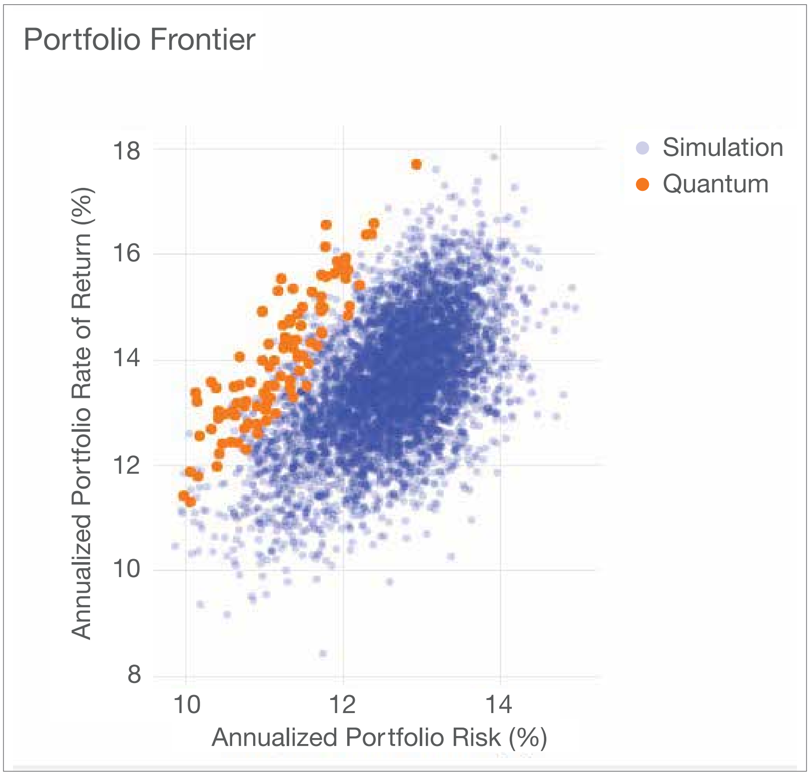
Portfolio optimization is an iterative procedure for selecting the best assets from a pool of possibilities. This problem necessitates the use of approximation tactics by the algorithm, as well as a traversal of all hypotheses to find the nearest solution. Such iterative computations are resource and time intensive; this is an excellent setting for a quantum computer, which can minimize computing time and improve the frequency with which portfolio optimization can be carried out. For certain types of energy landscapes reflecting a certain class of problems, quantum solvers such as hybrid classical-quantum solvers on quantum annealers and quantum-inspired classical optimization algorithms (QIO) can increase solution quality while reducing the run-time.

The following section captures the results of our portfolio optimization experiments.

6.

Experiment & Results

	Classical Approach - Monte Carlo Simulation	Quantum Annealing-based Solution
Infrastructure	16 GB RAM, Windows Machine	D-Wave 2000Q Hybrid Solver
Dataset	Stocks Daily Return Data of 30 Dow Jones stocks over 5 years	
Run-time	13.69 s	0.33 s
Portfolio Return (Annualized)	16.01%	17.71%
Portfolio Risk (Annualized)	12.55%	12.93%



The above diagram compares the portfolios built by Monte Carlo and the Quantum Optimization solutions. Amber dots above represent the portfolios built by Quantum Optimization solution and blue dots represent the result of the Monte Carlo simulation-driven portfolio. As per the results, Quantum Optimization delivers portfolios with better risk-return profiles than the Monte Carlo-based classical optimization solution.

7.

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Authors

Rohit Patel

AVP – Data Science and Quantum Computing

Rohit has 11+ years of experience in IT industry with 6+ years of experience in Data Science. He holds a B.Tech. in ECE and PGPM degree in Finance. He has been co-leading the Quantum Computing practice at Mphasis NEXT Labs for 2+ years. He has developed solutions in AI and Quantum Computing in areas of Logistics, Life Sciences and Process Optimization, among others.

Ashutosh Vyas

Senior Data Science Manager – Mphasis NEXT Labs

Ashutosh has 6+ years of experience in the data science domain. He has worked on multiple projects of pattern recognition, time series forecasting, regression modelling, NLP, classification and optimization in Life Sciences, Finance, FMCG and Media domains. He completed his MTech in 2015 from IIIT-B. He has expertise in Bayesian methods of machine learning and had been working in quantum ML and quantum optimization from the past 2 years and has developed multiple algorithms in image classification, and anomaly detection domain using quantum systems that leverage quantum gates and quantum annealer to process information and learn the patterns. At Mphasis, he works as a Senior Data Science Manager with an ethos of developing customer-centric and robust solutions.

Narayan Mishra

Assistant Manager – Data Science

Narayan Mishra has 3+ years of professional experience. He completed his Master of Technology from IIT Kanpur in Industrial & Management Engineering. He is proficient in Operations Research and Statistical Modelling. He is interested in various Machine Learning & Deep Learning areas like product recommendations, natural language processing, and computer vision. He has also been actively involved in building a production scale recommendation system for a large organization. He is exploring workflow management platforms for data engineering pipelines to programmatically author, schedule, and monitor workflows. He has a keen interest in Quantum Computing and Privacy-Preserving Machine Learning.

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For more information, contact: marketinginfo.m@mphasis.com

USA
Mphasis Corporation
41 Madison Avenue
35th Floor, New York
New York 10010, USA
Tel: +1 (212) 686 6655

UK
Mphasis UK Limited
1 Ropemaker Street, London
EC2Y 9HT, United Kingdom
T : +44 020 7153 1327

INDIA
Mphasis Limited
Bagmane World Technology Center
Marathahalli Ring Road
Doddanakundhi Village, Mahadevapura
Bangalore 560 048, India
Tel.: +91 80 3352 5000

www.mphasis.com

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